

if  $C$  and  $K_0'$  both positive we used

$$m - K_0' = \frac{C}{2[K_0' - (2C)^{1/2}]} \quad (4)$$

This expression was obtained as an approximation to the smallest value of  $m$  that allowed  $K$  to fall to zero on  $-a < P < 0$ , assuming  $C \ll K_0'^2$ . The condition that  $K$  drop continuously to zero on  $P < 0$  may be regarded as an 'instability' condition. It is satisfied automatically whenever  $K_0' > 0$  and  $C < 0$ . It is not considered essential for the purpose of extrapolating on  $P > 0$ , but in the absence of any other guidance, it seemed to be a reasonable criterion for relating the two adjustable parameters, say  $m$  and  $C$ , when  $K_0'$  and  $C$  are both positive. The idea that  $m$  should be near the smallest value that allows this instability follows from the feeling that the condition  $m > K_0'$ , needed to avoid a singularity on  $P > 0$  in this case, is likely to give an  $m$  that is already too large to be a correct limiting value of  $dK/dp$  as  $p \rightarrow \infty$ .

When  $K_0' < 0$ , as for vitreous silica, many formulas including those of Murnaghan and Keane necessarily predict an instability ( $K \leq 0$ ) on  $P > 0$ . Although this may not be a great catastrophe, and could even be represented as advantageous (because an actual material with  $K_0' < 0$  could be presumed to undergo a phase transition, through which the extrapolation should not be continued analytically), it is interesting to note that the present formula allows such an instability to be avoided by choosing a sufficiently high positive value for  $C$ . This is illustrated in Figure 1, which shows  $K/K_0$  versus  $P$  for three different values of  $C$  with  $K_0' = -6.5$  and  $m = 1$ . The value  $K_0' = -6.5$  applies to vitreous silica [McSkimin as cited by Anderson, 1961].

COMPRESSION EQUATION

The next task is to relate the volume  $v$  to the pressure, subject to equation 2 and the definition of the bulk modulus

$$K = -v dp/dv \quad (5)$$

$$v = \left\{ \frac{a}{mP^2 + (1 + A + am)P + a} \right\}$$

$$\cdot \left[ \frac{4am + 2mP[(q)^{1/2} + (1 + A + am)]}{4am - 2mP[(q)^{1/2} - (1 + A + am)]} \right]^{(1+A-am)/(q)^{1/2}} \quad (9)$$

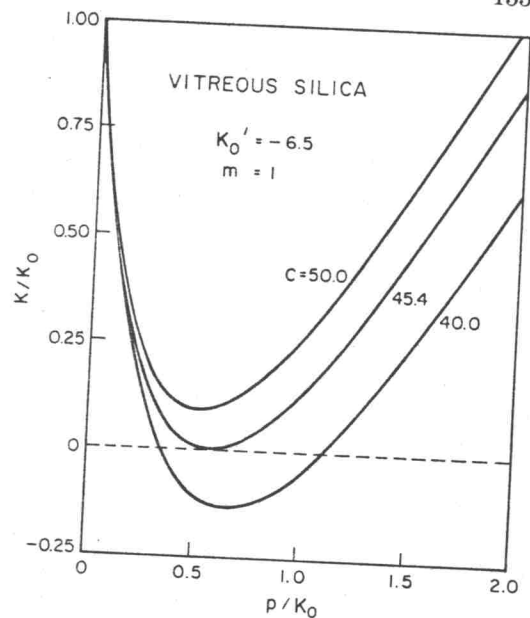


Fig. 1. Determining the value of  $C$  that ensures reasonable behavior of  $K$  on  $p > 0$  for the anomalous case, vitreous silica ( $K_0' < 0$ ) (see text).

Let  $V = v/v_0$ . Then

$$K/K_0 = -V dP/dV \quad (6)$$

As an abbreviation in equation 2, let  $A = a(K_0' - m)$ . Then the integral of equation 2 is

$$\frac{K}{K_0} = -V \frac{dP}{dV} = 1 + A + mP - \frac{aA}{P + a} \quad (7)$$

where the constant of integration has been determined to make  $K = K_0$  at  $P = 0$ . From (7)

$$V = \exp \left[ - \int \left( \frac{dP}{1 + A + mP - \frac{aA}{P + a}} \right) \right] \quad (8)$$

The evaluation of the integral in the expression above, subject to  $V = 1$  when  $P = 0$  (Given in Appendix B), gives us